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Recognition of ring pairs in the data analysis of Cherenkov detectors

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Abstract

We present a new algorithm for the identification of ring pairs in Cherenkov detectors. The approach combines linear regression methods for the construction of an appropriate reference quantity, methods from statistics for the evaluation of hypotheses and Monte Carlo simulation runs for the tuning of threshold levels.

Keywords: Ring pair recognition; Linear regression; Hypothesis testing; Tuning by simulation

1. Introduction

Evaluation of data from high energy physics experiments in many cases consists of the recognition of patterns out of a huge amount of (prepocessed) data points. In ring-image Cherenkov (RICH) detectors, each sufficiently fast charged particle generates a ring-shaped trace which is to be recognized from the measured data. These contain among "good" data also a lot of artificial effects prohibiting a fast pattern recognition code. Because of the large amount of data, evaluation and interpretation requires big supercomputers and efficient algorithms.

For the detection of rings, direct approaches like scanning the whole domain and counting points on the corresponding circle line may be considered as well as alternatives using, e.g., the Hough transform or neural algorithms (see, e.g., [2, 3]). For the fitting of the centers and radii, more or less sophisticated methods may be applied [1, 4]. In [5] an algorithm for the recognition of weakly populated rings has been proposed.

In the present paper we address a different problem within the same setting, the resolution of pairs of rings the centers of which differ by a small fraction of the radius only. The appearance of such "double rings" is a typical situation in RICH detectors, and it is important to have an efficient numerical tool to identify them.

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2. A reference parameter

Our aim is to assign for each "event" a value .true. or .false. to the hypothesis: "This event represents a double ring", and to estimate the reliability. An event is here a collection of points in $\mathbb{R}^2$ which are supposed to lie (up to slight deviations) on a circle or on one of two circles slightly shifted against one another. The fixed radius R and a reasonable estimate for the center of the (double) circle are supposed to be known. (For an estimate of these quantities see, e.g., [1].)

The method consists of constructing a reference parameter and to couple the evaluation of the hypothesis to a certain threshold. Suppose the set $E := \{(x_i, y_i),\ i = 1, \ldots, N\}$ represents an event, and (a_0, b_0) is an initial guess for the center. Under the assumption of one circle only, the "best" estimate for the center can be obtained by minimizing the defect of the following equations:

$$R^2 = (x_i - a_0 - \Delta a)^2 + (y_i - b_0 - \Delta b)^2,\ i = 1, \ldots, N.$$

The estimate for the center is then given by $(\hat{a}, \hat{b}) := (a_0, b_0) + (\Delta a, \Delta b)$. Since Δa and Δb are supposed to be small, we neglect quadratic (or higher order) terms. Define the vectors $\boldsymbol{\eta}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ by $\eta_i = R^2 - (x_i - a_0)^2 - (y_i - b_0)^2$, $\alpha_i = x_i - a_0$, $\beta_i = y_i - b_0$. The best values for Δa and Δb are then given by the requirement that the vector $\boldsymbol{\eta} + 2\Delta b \boldsymbol{\beta}$ is orthogonal to $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. This yields the linear system for $(\Delta a, \Delta b)$:

$$(\boldsymbol{\eta}, \boldsymbol{\alpha}) = -2\Delta a(\boldsymbol{\alpha}, \boldsymbol{\alpha}) - 2\Delta b(\boldsymbol{\beta}, \boldsymbol{\alpha}), \quad (\boldsymbol{\eta}, \boldsymbol{\beta}) = -2\Delta b(\boldsymbol{\alpha}, \boldsymbol{\beta}) - 2\Delta b(\boldsymbol{\beta}, \boldsymbol{\beta})$$

which can be solved easily. By a shift of the coefficients, we may assume that $(\hat{a}, \hat{b}) = (0, 0)$.

A good measure of how close the data points are to the ring is the functional

$$M(E, \Delta a, \Delta b) := \sum_{i=1}^{N} (R^2 - (x_i - a_0 - \Delta a)^2 - (y_i - b_0 - \Delta b)^2)^2.$$

As a crucial criterion for the double ring recognition turns out the comparison of $M(E,.)$ with the value minimized under the assumption that any of the N points lies on one of the two circles with radii R and centers $\varepsilon_1 \boldsymbol{\omega}$ and $-\varepsilon_2 \boldsymbol{\omega}$ with an unknown unit vector $\boldsymbol{\omega}$ and small positive quantities ε_1, ε_2. Let us first find out the optimal values for $\varepsilon_1, \varepsilon_2$ under the assumption that $\boldsymbol{\omega} = (1, 0)^{\mathrm{T}}$. We have to minimize the defect of the equations

$$R^2 = (x_i - \varepsilon_1 p_i + \varepsilon_2 q_i)^2 + y_i^2, \quad i = 1, \ldots, N.$$

Here, p_i (resp. q_i) is the indicator whether (x_i, y_i) belongs to the first circle (resp. to the second one), i.e., $p_i, q_i \in \{0, 1\}$, and $p_i + q_i = 1$. (Of course, p_i and q_i are not known in advance.) As before, neglecting quadratic terms of ε_i, the optimal quantities follow from the solution of the linear system of equations

$$(\boldsymbol{\eta}, \boldsymbol{\alpha}) = -2\varepsilon_1(\boldsymbol{\alpha}, \boldsymbol{\alpha}) - 2\varepsilon_2(\boldsymbol{\beta}, \boldsymbol{\alpha}), \quad (\boldsymbol{\eta}, \boldsymbol{\beta}) = 2\varepsilon_1(\boldsymbol{\alpha}, \boldsymbol{\beta}) - 2\varepsilon_2(\boldsymbol{\beta}, \boldsymbol{\beta}),$$

where now $\boldsymbol{\eta}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ are defined by $\eta_i = R^2 - x_i^2 - y_i^2$, $\alpha_i = -2x_i p_i$, $\beta_i = 2x_i q_i$. p_i and q_i are determined by inserting the solution into the corresponding minimizing functional

$$\tilde{M}(E, \varepsilon_1, \varepsilon_2) := \sum_{i=1}^{N} (R^2 - (x_i - \varepsilon_1 p_i + \varepsilon_2 q_i)^2 - y_i^2)^2.$$

It follows easily that $\tilde{M}$ becomes minimal if $p_i = 1$ for $\eta_i x_i < 0$ and $p_i = 0$ else. This condition generalizes to arbitrary fixed unit vectors ω as follows: $p_i = 1$ if $\eta_i(\omega, (x_i, y_i)^{\mathrm{T}}) < 0$ and $p_i = 0$ else.

3. Fitting the direction

We denote $\tilde{M}_w(E, \omega)$ the value of $\tilde{M}(E,.)$ optimized along the direction ω. Fig. 1 shows the probability distributions of $\tilde{M}_w$ obtained from Monte Carlo simulations of one-circle and double-circle events. In both cases, the points are scattered within $\pm 1\%$ of the radius along the circle lines. In the double-circle case, the points are randomly associated to one of the rings which are shifted 5% of the radius in direction $\pm \omega$ for known ω. Fig. 1 shows that both events separate quite well. A threshold level of 7 would separate single rings and pairs of rings with a certainty of approximately 99%.

The problem is that of course we do not know the direction ω in advance. Since well-known techniques like the evaluation of certain moments turns out not to give good results, we scan the domain $\{(\cos\phi, \sin\phi), \phi \in [0, \pi)\}$ of possible unit vectors: We calculate $\tilde{M}_\omega(E, \omega_k)$ for the vectors

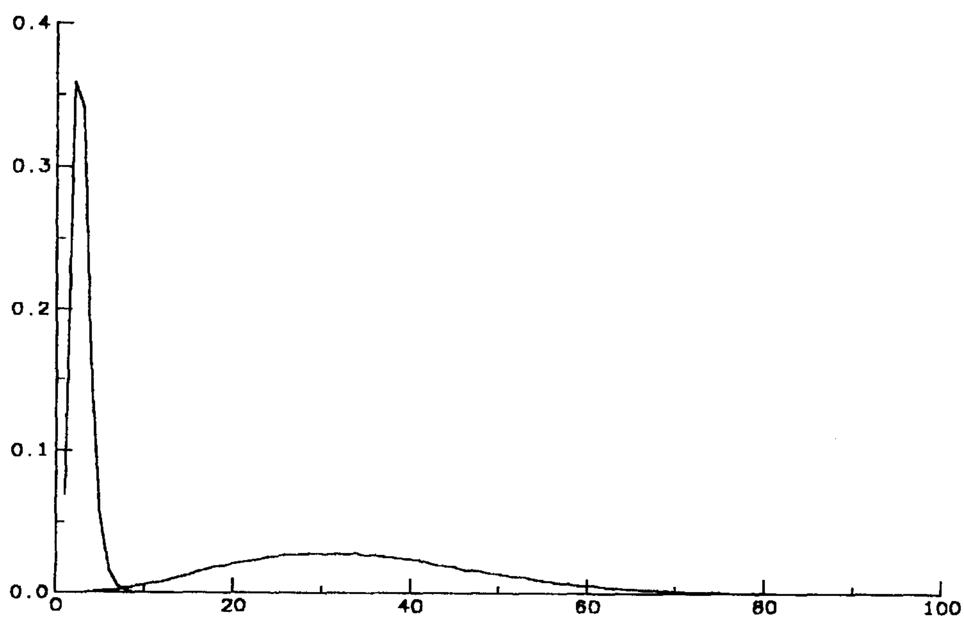

Fig. 1. Distribution of $\tilde{M}$ for single rings and for ring pairs.

$\omega_k = (\cos k\pi/K, \sin k\pi/K)$, $k = 1, \dots, K$ and choose the minimal value $(\tilde{M}_\omega)_{\min} := \min \tilde{M}_\omega(E, \omega_k)$ as reference quantity. (In some cases, the fraction $(\tilde{M}_\omega)_{\min}/(\tilde{M}_\omega)_{\max}$ yields better results.)

Figs. 2 and 3 show the distributions for $K = 4$ and $K = 16$. Statistics based on $K = 4$ turn out to be quite poor. On the other hand, increasing K from 16 to 32 only has a marginal effect (0.2%). Therefore, a good compromise between short calculation times and good quality results might be $K = 16$.

4. Tuning the threshold level

For short, we denote $\tilde{M}$ one of the two reference quantities mentioned at the end of the previous section. Fig. 3 shows the distribution of $\tilde{M}$ both for one-circle and double-circle events. The remaining problem is now that of accepting or rejecting the hypothesis of a double ring on the basis of the measured value $\tilde{M}(E)$. Denote by $(N = 1)$ the event "double ring" and by $(N = 0)$ the event "single ring". Bayes' formula yields

$$P(N = 0 | \tilde{M}(E) \geqslant X) = \frac{P(\tilde{M}(E) \geqslant X | N = 0)\ p(N = 0)}{P(\tilde{M}(E) \geqslant X)}$$

and

$$P(N = 1 | \tilde{M}(E) \leqslant X) = \frac{P(\tilde{M}(E) \leqslant X | N = 1)\ (N = 1)}{P(\tilde{M}(E) \leqslant X)}.$$

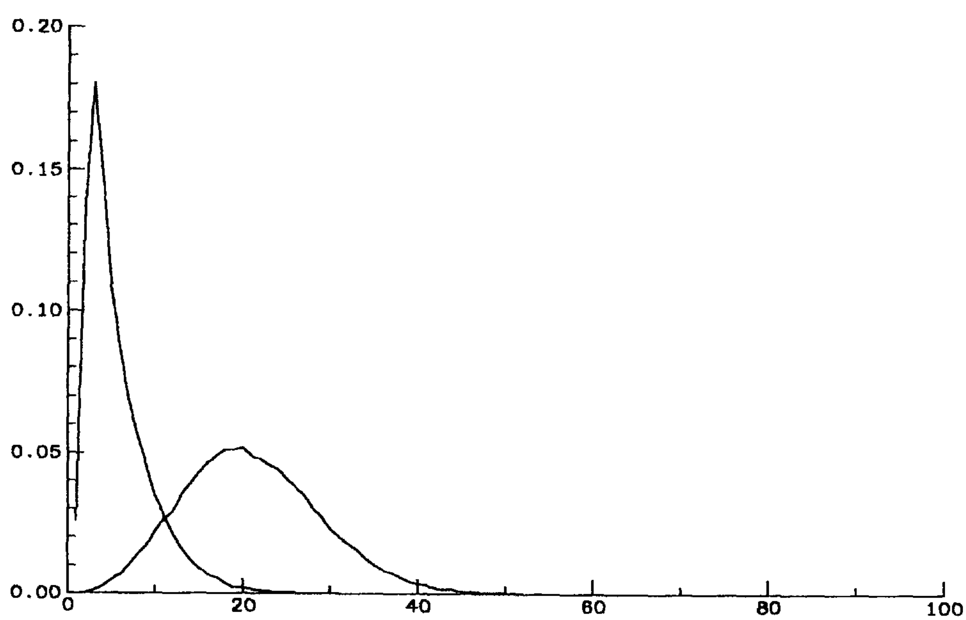

Fig. 2. Distributions of $\tilde{M}_\omega(\omega_{\min})$ for single rings and for ring pairs when scanning four directions.

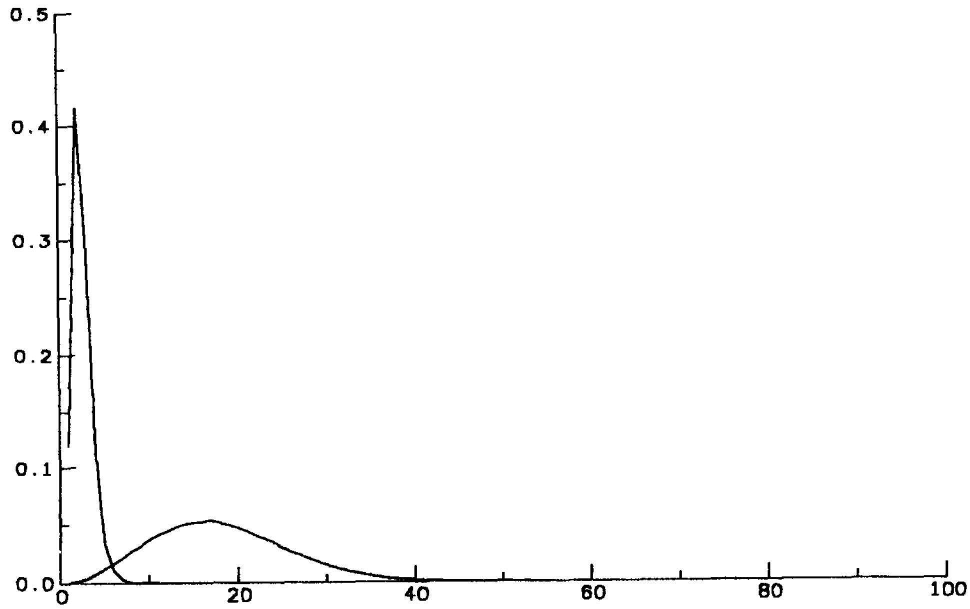

Fig. 3. Distributions of $\tilde{M}_\omega(\omega_{\min})$ for single rings and for ring pairs when scanning 32 directions.

The distributions of $\tilde{M}$ for $(N = 0)$ and $(N = 1)$ and with them the optimal threshold level can be obtained from simulation runs. They have to be generated separately for each possible configuration like number NP of points related to each event, degree σ of scattering of points around a circle line, and the minimal distance ΔP between the centers of a ring pair expected be resolved. (In Figs. 1–3 we used NP $= 14$, $\sigma = 1\%$ of radius, $\Delta P = 10\%$.) We assume the value $P(N = 0)$ to be known in advance. (Notice that this value can be obtained approximately by comparing the distributions of Fig. 3 with the one obtained from the experiment.) We assumed $P(N = 0) = 0.5$. Bayes' formula allows to run a short program calculating the threshold level X_{thr} for which $P(N = 0|\tilde{M}(E) \geqslant X_{\text{thr}}) = P(N = 1|\tilde{M}(E) \leqslant X_{\text{thr}} =: r{\cdot}r$ represents the reliability of the algorithm.

5. Some performance results

The algorithm proposed above has been evaluated for several distances ΔP and fluctuation parameters σ. (The other parameters have been chosen an in the previous section. The results (more precisely: the reliability r of the code) are presented in Table 1. They are based on a threshold level for the variable $\tilde{M}_\omega(\omega_{\min})/\tilde{M}_\omega(\omega_{\max})$ which yields slightly better results than that of $\tilde{M}_\omega(\omega_{\min})$.

As it turns out, a high recognition rate is restricted to values of ΔP sufficiently larger than σ. This is natural, since large deviations of the points from the circle line prohibit the resolution of double rings close together. On the other hand, results also grow worse for ΔP too large. This seems to be due to the fact that the linearization assumptions are no longer valid. A non-linear approach is possible in this case but certainly requires much larger calculation times.

Table 1
Reliability of code

	σ			
ΔP	0.01	0.02	0.04	0.08
0.04	0.788	—	—	—
0.08	0.961	0.786	—	—
0.12	0.986	0.909	0.662	—
0.16	0.989	0.952	0.779	—
0.20	0.987	0.965	0.851	—
0.24	0.984	0.967	0.891	0.652
0.28	0.977	0.964	0.911	0.711
0.32	0.968	0.958	0.918	0.756
0.36	0.957	0.949	0.918	0.789
0.40	0.945	0.939	0.914	0.811

Parameters interesting for the evaluation of data RICH detectors are in the region $\sigma < 6\%$ and $\Delta P > 10\%$. For most of these values, the uncertainty is much less than 10%. As is shown in Fig. 3, the distribution of $\tilde{M}$ is rapidly decreasing in the critical region. Thus by excluding a small fraction of events as indistinguishable by this algorithm, the reliability can be increased even more. In the examples above, the deviations of the points from the circle lines are assumed to be equidistributed in the interval $(-\sigma, +\sigma)$. A more realistic assumption is that of a Gaussian distribution which is handled in the following section.

6. Application to "realistic" data

We wanted to test the algorithm in a more realistic setting and to find out the limit of the range of applications for which our code yields satisfactory results. To this aim we used a code for the generation of test data which has been developed by the group of P. Glaessel at the Max Planck Institute in Heidelberg and which is intended to produce data with a statistics close to that produced in high energy experiments at CERN. This procedure generates points placed randomly with a Gaussian distribution around rings. We used a standard deviation of 5% of the ring radius. In order to compare with the results of the previous sections, we assumed a total of 14 points on each (double) ring. In the case of a double ring, an information which is not available is the number of points on each of the two rings. We assumed a binomial distribution: the probability of k points on one and $14k$ points on the other ring is

$$\binom{14}{k}\left(\frac{1}{2}\right)^{14}.$$

Under these conditions, the single ring and double ring distributions corresponding to Fig. 3 have been generated using 700 000 and 490 000 samples, respectively. The distributions are shown in Fig. 4. The reliability of the hypothesis is about 67% and thus not satisfactory. One main reason is the

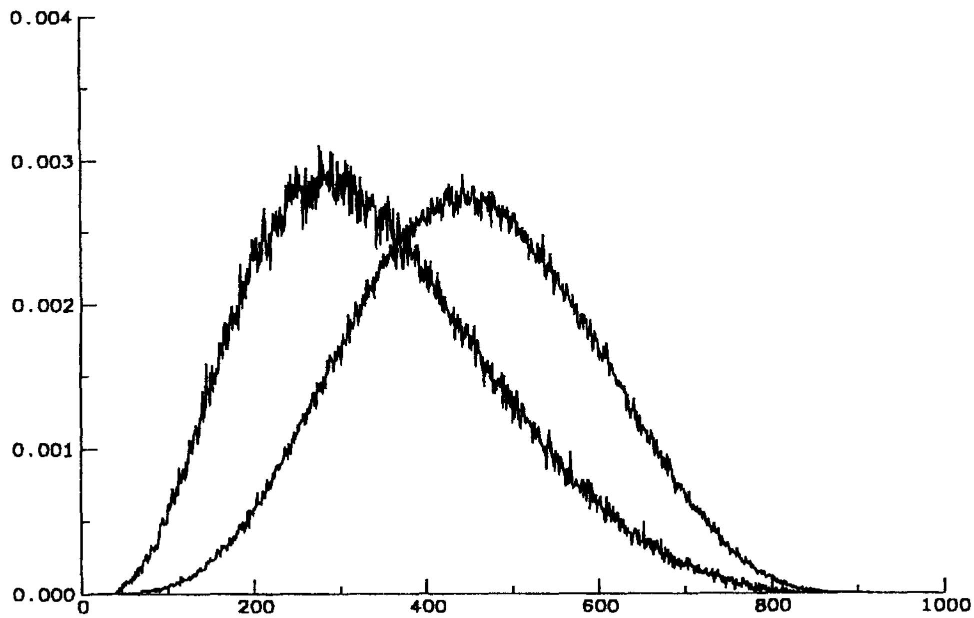

Fig. 4. Distribution for single rings and for ring pairs for "realistic" data.

lack of information about the random variable k: Obviously, the case $k = 0$ and $k = 14$ are not distinguishable from a single ring situation. As the inspection of the corresponding distributions shows, the same is true for $k = 1, \dots, 3$ and $k = 11, \dots, 13$. Another reason is that the distance of the ring centers is not fixed but a random variable which, with a certain probability, is close to zero.

One conclusion of this might be that the algorithm only separates double rings with lower deviations, larger ring distances or larger numbers of points. On the other hand, there are several ways to improve reliability: one may

- define a (small) percentage of events within the critical regime as "undecidable" thus increasing reliability of the other events;
- (and has to) use more realistic values for the probability of a double ring; smaller values lead to smaller threshold levels thus improving separation;
- (most important:) couple the data to the data produced by a second RICH detector, which indeed is done at CERN.

In practice, a definition of the final setting for the evaluation has to come through a careful study of the statistics produced by the data of the experiment.

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